

## GEOG401 Climatology: Global Energy Balance

---

- Radiation is emitted from all objects with a temperature above absolute zero (0 degrees Kelvin). The amount of radiation emitted by an object is determined by the Stefan Boltzmann law that states that the blackbody emission is  $I = \sigma T^4$ , where  $\sigma$  is the Stefan Boltzmann constant of  $5.67 \times 10^{-8} W m^{-2} K^{-4}$ , and  $T$  is temperature of the object in degrees Kelvin. Note that the actual amount of radiation emitted from an object also depends on the emissivity  $\epsilon$  of an object, for a blackbody, we assume that  $\epsilon = 1$ .
- Energy Balance refers to an equilibrium between the energy inputs and energy outputs. Hence, when solving for energy balance, it is advisable to set up equations as inputs = outputs.
- Three things control the global energy balance and surface temperature: (1) the amount of solar radiation reaching the top of the Earth's atmosphere, (2) the reflectivity of the planet (both surface and atmosphere), and (3) the greenhouse effect.

(1) **The sun** The sun emits shortwave radiation, with peak intensity of radiation in the visible spectrum near 0.5 micrometers. Radiation emitted from the surface area of the sun traverses space, with the intensity of radiation diminishing as a function of the inverse distance squared from the sun [called the inverse distance squared law, technically it drops off at  $4\pi$  times the distance squared]. If you know the temperature of the sun, its surface area, and the distance between the sun's surface and Earth's surface, you can geometrically deduce the amount of radiation reaching the top of the planet. This number is often referred to as the solar constant ( $S_o$ ), despite the fact that it does vary over time. For our purposes  $S_o = 1367 W m^{-2}$ .

### Hypothetical Earth Part 1: The black marble

Our first attempt assumes that the sole contributor to the energy balance of Earth is the solar radiation reaching the top of the Earth's atmosphere. In doing so we are assuming the planet absorbs all of this radiation (and reflects none of it), and that there is no greenhouse effect. Let's set up an equation that balances the inputs (the solar inputs) with the outputs (radiation emitted back to space from the Earth).

$$S_o \pi r^2 = \sigma T^4 4\pi r^2 \quad (1)$$

Note that the left hand side of the equation is multiplied by the surface area of a circle with a radius the the Earth, while the right hand side is multiplied by the surface area of the Earth. The differences in the surface are of a circle (illuminated by the sun, but spread over half the planet at a given time) and surface area of a sphere (over which radiation is emitted), allow us to simplify this equation and solve to  $T$ .

$$T = \sqrt[4]{\frac{S_o}{4\sigma}} \quad (2)$$

Plugging in the solar constant of  $1367 W m^{-2}$  into equation 2 yields an answer of  $T = 278K$ . A bit cooler than the real planet that has a temperature of  $288K$ .

### (2) Albedo

Of course, not all of the radiation the Earth receives from the sun is absorbed by the atmosphere or surface and contribute to the planet's temperature. Around 30 percent of the potential radiation from the sun is reflected back to space. We refer to the proportion of reflected shortwave radiation as albedo  $\alpha$ . Let's rewrite equation 1 now including the albedo concept.

## Hypothetical Earth Part 2: The blue and white marble

$$S_o(1 - \alpha)\pi r^2 = \sigma T^4 4\pi r^2 \quad (3)$$

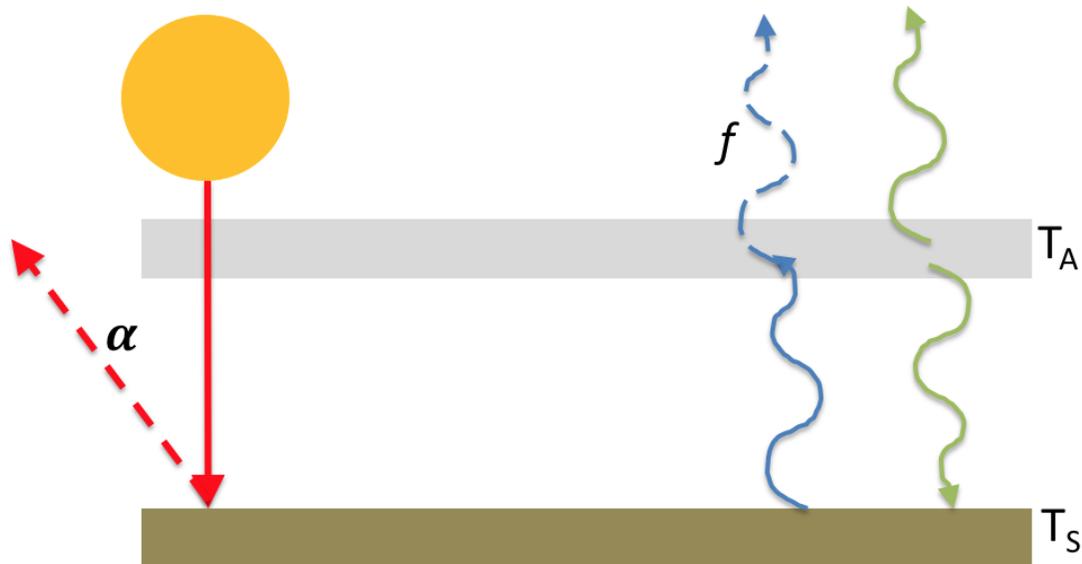
Immediately, you should note that there is a lower number on the input side, and that this should translate into less radiation emitted back to space to balance the energy budget - hence a lower temperature. If you plug in a value for  $\alpha$  of 0.3, you'll find that we have a value for  $T$  of 255K. Terribly cold. This is the temperature of a blue and white marble with no atmosphere located 150 million kilometers from the sun. Aren't you glad to you don't live there?

### (3) The greenhouse effect

The greenhouse effect refers to the discrimination of certain molecules absorbing radiation of different wavelengths. Most of the Earth's atmosphere is composed of Nitrogen and Oxygen which are inert and not greenhouse gases. By contrast, molecules that are 3 or more atoms including water vapor, carbon dioxide, methane, and ozone are greenhouse gases. Most of these gases absorb radiation emitted in the longer wavelengths, including that emitted by the Earth itself. Ozone is a bit of an outlier as it absorbs both UV shortwave radiation (our ozone layer) as well as long wavelengths.

### Hypothetical Earth Part 3: Adding a greenhouse effect

We can simplify this process by assuming the atmosphere is a single layer (not true), and that a certain fraction of radiation emitted from the Earth's surface can escape back to space without being trapped by the greenhouse effect. We will also ignore the shortwave radiation absorbed by ozone in the stratosphere. Let's refer to the figure below:



In this figure, a certain fraction ( $f$ ) of the longwave radiation emitted from the Earth's surface (blue) escapes back to space without being trapped by the Earth's atmosphere. The Earth's atmosphere radiations both back to space and to the Earth's surface. The downward longwave radiation emitted from the atmosphere to the surface is the greenhouse effect.

We need to set up two equations, one that shows the balance at the top of the atmosphere

$$\frac{S_o(1 - \alpha)}{4} = \sigma T_A^4 + f\sigma T_S^4 \quad (4)$$

Where  $T_S$  is the temperature of the surface, and  $T_A$  is the temperature of the atmosphere. Next, we write an equation for the surface, note that now we have a secondary input which is the radiation emitted from the atmosphere down to the surface. This is the greenhouse effect.

$$\frac{S_o(1 - \alpha)}{4} + \sigma T_A^4 = \sigma T_S^4 \quad (5)$$

I'll save you the math, but since we have two equations and two unknowns, we can actually solve for  $T_S$ .

$$T = \sqrt[4]{\frac{S_o(1 - \alpha)}{4 * (0.5 + f/2) * \sigma}} \quad (6)$$

If we assume here that  $f=0.2$ , we end up having a surface temperature  $T_S$  that is 289K, which is about what we observe.